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STUDY OF THE INFLUENCE OF NONLINEARITIES ON PARAMETRIC OSCILLATIONS OF ELECTROMECHATRONIC SYSTEMS

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ДОСЛІДЖЕННЯ ВПЛИВУ НЕЛІНІЙНОСТЕЙ НА ПАРАМЕТРИЧНІ КОЛИВАННЯ В ЕЛЕКТРОМЕХАТРОННИХ СИСТЕМАХ

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In the article The analysis of parametric oscillations in electromechatronic systems and the influence of nonlinearities on them caused by physical properties of the elements and design features of the system.

It has been shown that Real electromechatronic systems are characterized by the presence of various nonlinearities. Factors that render a mechanical system nonlinear include nonlinear forces arising during the operation of such a system and its variable parameters. It has been established that parametric oscillations, caused by periodic or quasi-periodic changes in the system's parameters over time, occupy a special place among the dynamic phenomena occurring in electromechatronic systems.

The article discusses the effect of parametric resonance, which is a manifestation of dynamic instability in a system, where small disturbances can lead to significant changes in the system's motion. Consequently, any random disturbance over a sufficiently long period of time can lead to emergency consequences. Nonlinear factors in a mechanical system with periodically changing parameters manifest themselves primarily in zones of parametric resonance.

The authors present the concept of self-oscillations, which can occur in nonlinear systems in the absence of a periodic disturbing force, and, as an example, consider quasi-harmonic frictional self-oscillations. Suppressing frictional self-oscillations is a crucial engineering challenge, as they complicate the precise stopping of a working element and disrupt the smoothness of its movements.

A unified structural diagram for any type of electromechanical electric drive system, its parameters, and a mathematical model are provided.

The electromechanical system was simulated in the MATLAB/Simulink environment in accordance with the structural diagram with the given parameters. The processes of development of self-oscillations in the considered circuit at a given speed for three variants of the studied load characteristics are obtained.

The dependences of the velocities of the first and second masses, the elastic force and resistance on the second mass, and the dynamic force on time are presented. It is established that the implementation of computer - integrated adaptive and robust control systems, taking into account the presented studies of nonlinear effects and parametric oscillations, ensures the coordination of the dynamic modes of the electromechatronic system with the requirements of a specific technological process. This reduces the influence of unwanted oscillatory modes and improves the stability and accuracy of actuator control.

Key words: *parametric oscillations, electromechatronic systems, nonlinearity, mathematical model, disturbance, self-oscillation, synthesis, computer-integrated control system, adaptive control, robust control.*

Introduction. Modern electromechatronic systems are complex dynamic objects combining electrical, mechanical, electronic, and information subsystems. They are widely used in automated production facilities, robotic devices, electric drives, precision positioning systems, and power plants. The increasing level of integration and complexity of the structures of such systems leads to increased demands on the quality of their dynamic characteristics, stability, and operational reliability under variable loads and external disturbances [1, 2].

Real electromechatronic systems are characterized by the presence of various nonlinearities, caused by the physical properties of the elements and the design features of the system.

Factors that render a mechanical system nonlinear include nonlinear forces arising during the operation of such a system and its variable parameters [3].

The nature of nonlinear forces can be different [1]:

1) restorative forces when elastic characteristics deviate from Hooke's law;

2) dissipative phenomena such as «dry» (Coulomb) friction $M_{DF} = -|M_{DF}| \, sign\omega$;

3) nonlinear forces associated with the design features of the mechanical system (gaps, limiters, clamps, stops, special couplings), in which the transmitting moment depends on the magnitude of the elastic deformation of the coupling elements – $M_E(\Delta\varphi)$;

4) forces arising in mechanisms with a nonlinear position function, when linearization in the vicinity of the current value of the rotation angle is impossible due to large angular deviations of the driving elements (systems with a clearly variable gear ratio; for example, clutch slip).

These factors have a significant impact on the dynamic behavior of the system and can lead to the emergence of complex oscillatory modes.

Nonlinear characteristics of mechanical systems are often deliberately used to achieve desired dynamic effects (vibration-impact mechanisms). However, nonlinearity often leads to undesirable effects (impact phenomena in gaps, frictional self-oscillations, and others) [4].

Parametric oscillations, caused by periodic or quasi-periodic changes in system parameters over time, occupy a special place among the dynamic phenomena occurring in electromechatronic systems. Such oscillations can arise as a result of modulation of stiffness, mass, inductance, or other parameters dependent on the operating mode or external influences. Parametric resonances can cause a sharp increase in oscillation amplitude, loss of stability, and deterioration of the system's performance [5].

The presence of nonlinearities significantly complicates the analysis of parametric oscillations, as it changes the conditions under which they arise, creating additional instability zones and a variety of motion modes. Nonlinear effects can both limit the growth of oscillation amplitude and contribute to the development of chaotic processes, which negatively impacts control accuracy and the durability of system components. Therefore, the use of linear models is often insufficient to adequately describe the real dynamics of electromechatronic systems.

The objective to analyze and evaluate the influence of nonlinear characteristics of electromechatronic system elements on the conditions for the occurrence, development, and parameters of parametric oscillations, as well as to determine their impact on the dynamic stability and operating modes of the system in order to improve the modeling efficiency and reliable operation of electromechatronic systems.

Research results. There are a significant number of mechanisms with a nonlinear position function *P,* which relates the output x and input φ coordinates: $x = P(\varphi)$. These include piston pumps and compressors, guillotine shears, and others. In these cases, it becomes necessary to detect, to a first approximation, the distortions arising from drive vibrations.

The initial data for constructing a mathematical model of the system are the equations of motion and dynamic characteristics of the engine:

$$J\frac{d\omega}{dt} + \frac{\omega^2}{2}\frac{dJ(\varphi)}{d\varphi} = M - M_C; \quad \omega = \omega_0\left[1 - \gamma\left(M + T_e\frac{dM}{dt}\right)\right]. \tag{1}$$

After linearization in the vicinity of the current phase angle and elementary transformations, we obtain a second-order differential equation with variable coefficients of this type [1, 2]:

$$\frac{d^2\omega}{dt^2} + 2n(t)\frac{d\omega}{dt} + K^2(t)\omega = W(t), \tag{2}$$

where $2n(t) = \frac{1}{T_e} + 2\omega_*\frac{J'_*}{J_*}$;

$$K^2(t) = \frac{1}{J_*}\left[\frac{1}{\gamma T_e\omega_*} + \frac{J'_*\omega_*}{T_e} + \frac{3}{2}\omega_*^2 J''\right]; \tag{3}$$

$$W(t) = -\frac{1}{J_*}\left(\frac{M_C}{T_e} + \frac{dM_C}{dt} + \frac{J'_*\omega_*^2}{2T_e} + \frac{J''\omega_*^3}{2}\right);$$

$$J' = dJ/d\varphi; \; J'' = d^2J/d\varphi^2 .$$

$K^2(t)$ can be considered as a variable «natural» frequency, and $W(t)$ as a disturbance function.

Asterisks (*) correspond $\varphi = \varphi_* = \omega_* t$ to the average value of angular velocity ω_* in steady-state conditions. This is correct for small unevenness coefficients, typical for many devices.

When $J(\varphi) = const$ we have $J' = J'' = 0$ the coefficients of equation (2) become constant.

The solution of the homogeneous equation according to (2) is proposed in [1, 2]:

$$\ddot{\omega}+2n(t)\dot{\omega}+K^2(t)\omega=0. \qquad (4)$$

By changing variables, we move on to a differential equation in which the term containing the first derivative of the coordinate is missing. For this, we use the substitution

$$y=\omega\cdot\exp\left[\int_0^t n(t)\,dt\right], \qquad (5)$$

Then

$$\left.\begin{aligned}\omega&=y\cdot\exp\left[-\int_0^t n(t)\,dt\right]\\ \dot{\omega}&=(\dot{y}-ny)\exp\left[-\int_0^t n(t)\,dt\right]\\ \ddot{\omega}&=\left(n^2y-2n\dot{y}-\dot{n}y+\ddot{y}\right)\exp\left[-\int_0^t n(t)\,dt\right]\end{aligned}\right\} \qquad (6)$$

After substituting (6) into (4) we obtain

$$\ddot{y}+p^2(t)y=0, \qquad (7)$$

where $p^2(t)=K^2(t)-n^2(t)-\dot{n}(t)$.

Moreover, in this class of problems p^2/n^2 it differs little from one.

According to the "conditional oscillator" method, the solution is sought in the form [1, 2]

$$y=B(t)\cos\Phi(t). \qquad (8)$$

Differentiating (8) twice, we obtain

$$\ddot{y}=\left(\ddot{B}-B\dot{\Phi}^2\right)\cos\varphi(t)-\left(2\dot{B}\dot{\Phi}+B\ddot{\Phi}\right)\sin\varphi(t). \qquad (9)$$

Solution (8) is presented as a product of two unknown functions, which gives the right to connect these functions with one additional condition. If we accept

$$2\dot{B}\dot{\Phi}+B\ddot{\Phi}=0, \qquad (10)$$

then the dependence (9) will look like this

$$\ddot{y}=\left(\ddot{B}-B\dot{\Phi}^2\right)\cos\Phi(t). \qquad (11)$$

Next, we find the relationship between the functions *B*, Φ and the «natural» frequency *p*(*t*). To do this, it is sufficient to substitute (8) and (11) into (7). From where

$$\Omega^2-\frac{\ddot{B}}{B}=p^2, \qquad (12)$$

where $\Omega=\dot{\Phi}$.

The additional condition (10) is a first-order differential equation with separable variables with respect to *B* and Ω.

$$\frac{dB}{B}=-\frac{1}{2}\frac{d\Omega}{\Omega}, \qquad (13)$$

the solution of which is

$$B=A\sqrt{\frac{\Omega_0}{\Omega(t)}}, \qquad (14)$$

where $B(0)=A$; $\Omega(0)=\Omega_0$.

Now equation (3) takes the form

$$y=A\sqrt{\frac{\Omega_0}{\Omega(t)}}\cos\left[\int_0^t\Omega(t)\,dt+a\right]. \qquad (15)$$

Here *A* and *a* are determined by the initial conditions. After substituting (14) into (12) and making some transformations, we obtain

$$\ddot{Z}-0,5\dot{Z}^2+2\Omega_*^2e^{2Z}=2p^2(t), \qquad (16)$$

where $Z=\ln(\Omega/\Omega_*)$, Ω_* is a free parameter with the dimension of frequency.

Differential equation (16) corresponds to a certain oscillatory circuit (oscillator) with a «rigid» nonlinear characteristic. The perturbation is a function proportional to the square of the «natural» frequency. Since the variable *Z* can only be considered as an analogue of some deformation, such an «oscillator» is called conditional. It is sufficient to have a solution to the conditional oscillator equation (16) for (15) to transform into a calculated dependence determining the solution to the homogeneous equation (7). To obtain a solution to the original equation (4), it is necessary to address the initial variable ω, respectively, as in (5).

$$\omega = A\exp\left[-\int_0^t n(t)\,dt\sqrt{\frac{\Omega_0}{\Omega}}\cos\left(\int_0^t \Omega(t)\,dt + a]\right)\right] =$$
$$= A\exp\left[-\int_0^t n(t)\,dt - 0{,}5\left(Z - Z_0\right)\right]\cdot \qquad (17)$$
$$\cdot\cos\left(\Omega_0\int_0^t e^{Z(t)}dt + a\right),$$

When $|Z| < 1$ one can use the linearization of the coefficients of equation (16). In this case, the latter takes the form

$$Z + 4p_*^2 Z = 2\left(p^2 - p_*^2\right), \qquad (18)$$

where $p_* = \Omega_*$ is the average value of the function $p(t)$.

It should be noted that, according to (17), unlimited growth of ω will occur only due to the exponential factor, which in turn can reach infinitely large values at $Z \to \infty$. Since this effect is caused not by an external force, but by changes in the system's parameters, it is called parametric resonance. Parametric resonance is a manifestation of the dynamic instability of a system, when small disturbances can lead to significant changes in the system's motion. From (17), it follows that at, $A = 0$ respectively $\omega = 0$. However, sufficiently small initial conditions, under which $A \neq 0$, can $t \to \infty$ be obtained $\omega \to \infty$ through an increasing exponential factor. The practical importance of the concept of dynamic instability follows, firstly, from the fact that unstable motion is uncontrollable, meaning that any random disturbance over a sufficiently long period of time can lead to emergency consequences. Moreover, the dynamic model is far from completely equivalent to the physical original. Deviations caused by these inaccuracies can be viewed as disturbances, which, in the case of instability, will lead to significant distortions of the obtained solution.

The condition of dynamic stability is [1, 2]:

$$\lambda = \int_0^T n(t)dt > 0{,}5|\Delta Z|, \qquad (19)$$

where ΔZ is the difference between the minima of the function *Z,* separated by the period *T* ($T = 2\pi/p_*$); λ is the logarithmic decrement.

In the presence of a pulsation of the function $p^2(t)$ with frequency ω:

$$p^2(t) = p_*^2\left(1 - \varepsilon\cos\omega^* t\right), \qquad (20)$$

where ε is the pulsation depth,
then using (19), equation (18) can be represented in the following form:

$$\ddot{Z} + 4p_*^2 Z = 2\varepsilon p_*^2\cos\omega^* t\,. \qquad (21)$$

It is obvious that the «conditional oscillator» resonates at $\omega^* = 2p_*$, which corresponds to the main parametric resonance. Its development in accordance with (21) follows the law [1, 2]:

$$Z = -0{,}5p_* t\varepsilon\sin 2p_* t\,, \qquad (22)$$

Where

$$|\Delta Z| = |Z(t_2) - Z(t_1)| = \pi\varepsilon\,, \qquad (23)$$

Where $t_1 = \pi/4p_*$; $t_2 = t_1 + T$; $T = 2\pi/p_*$.

Substituting (23) into (19), we have:

$$\lambda > 0{,}5\pi\varepsilon\,, \qquad (24)$$

which is $\psi = 2\lambda$ equivalent to

$$\psi > \pi\varepsilon\,. \qquad (25)$$

Nonlinear factors in a mechanical system with periodically changing parameters manifest themselves primarily in parametric resonance zones. The critical oscillation frequency, around which a region of dynamic instability forms, now depends on the amplitude level of the oscillations.

$$\omega_{KP}^* = \frac{2}{j}K\left(A_0, A\right). \qquad (26)$$

Moreover, sufficient conditions (19) and (24) of the dynamic stability of the system are also functions of A_0, *A*, since the logarithmic decrement depends on the coefficient $n(A_0, A)$ [1, 2]

$$\lambda = \int_0^T n\left(A, A_0, t\right)dt\,.$$

In linear systems, all free oscillations are damped by dissipation. Oscillations in steady-state conditions can persist due to external periodic influences (forces). In nonlinear systems, however, established periodic oscillations can arise even in the absence of a periodic disturbing force. These are called self-oscillations. In a self-oscillating mechanical system, the action of dissipative forces is compensated by an external energy source controlled by nonlinear feedback loops dependent on the system's oscillations. In all cases, the onset of self-oscillatory conditions is preceded by a zone of dynamic instability.

As an example, let's consider quasi-harmonic frictional self-oscillations. They can be observed during the movement of supports, tables along sliding guides in metal-cutting workbenches, ingots along the hearth of a heating furnace, etc.

A simplified dynamic model of such systems is shown in Fig. 1, a. From element 1 moving at a constant speed, $V_0 > 0$ motion is transmitted through an elastic-dissipative element (C_{12}, ψ_2) to link 2, which may have a speed $V_0 > 0$ different from V_0. A friction force F_C is applied to link 2.

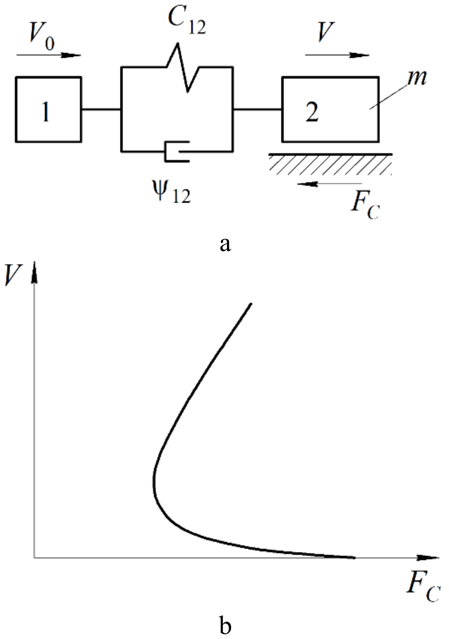

Fig. 1. Dynamic model (a) and dependence of friction force on speed (b) for mechanisms with «negative» viscous friction [1]

The differential equation under the conditions $V > 0$ has the form:

$$m\frac{d^2q}{dt^2}+\beta_{12}\frac{dq}{dt}+C_{12}q=-F_C\left(V\right) \qquad (27)$$

where m is the mass of element 2, kg;

C_{12} – coefficient of rigidity of the kinematic connection between 1 and 2, N/m;

β_{12} – coefficient of equivalent linear resistance (viscous friction, reflected by the dissipation coefficient ψ_{12}), Ns/m;

q – deformation of the elastic-dissipative connection (drive), m.

Dividing both parts (27) by m, we have

$$\frac{d^2q}{dt^2}+2n_0\frac{dq}{dt}+K_0^2q=-\frac{1}{m}F_C\left(V\right), \qquad (28)$$

where $2n_0=\beta_{12}/m$; $K_0^2=C_{12}/m$.

The coefficient n_0 can be considered as the coefficient of harmonic linearization of the dissipative forces of the drive, corresponding to the dissipation coefficient ψ_{12}.

The sliding speed consists of two components:

V_0 – speed set by the drive;

$\frac{dq}{dt}$ – the speed caused by vibrations.

Let us expand $F_C(V)$, which determines the friction force, into a Taylor series in powers of q [1, 2].

$$F_C\left(V_0+\dot{q}\right)\cong F_{C0}+h_1\dot{q}+h_2\dot{q}^2+h_3\dot{q}^3, \qquad (29)$$

where $F_{C0}=F\left(V_0\right)$; $h_i=\frac{1}{i!}\frac{d^iF_C}{dV^i}\left(V_0\right)$, $i=1,2,3$.

To determine the coefficients h_i, the static characteristic of friction is used (Fig. 1, b), taking into account (29), the differential equation (28) based on the harmonic linearization method can be represented as follows:

$$\psi=2n_0\frac{dq}{dt}+K^2q+ \\ +\frac{1}{m}\left[F_0+h_1\frac{dq}{dt}+h_2\left(\frac{dq}{dt}\right)^2+h_3\left(\frac{dq}{dt}\right)^3\right] \qquad (30)$$

After transformations, we have

$$2n_0+\frac{1}{m}\left(h_1+\frac{3}{4}h_3A^2\Omega^2\right)=0\,. \qquad (31)$$

Since the harmonic linearization coefficients are functions of three variables A, A_0, Ω, then, taking into account that $\Omega=K_0$ the amplitude of oscillations can be given as follows:

$$A=2\cdot\sqrt{-\frac{2n_0m+h_1}{3h_3K_0^2}}\,. \qquad (32)$$

The stability condition of this regime is determined by the inequality [1, 2]

$$\left[\frac{\partial n(A)}{\partial A}\right]_{A=A_i} > 0 . \qquad (33)$$

Since the left side of equation (31) is proportional to *n*(A), then differentiating it with respect to *A* and substituting it into (33), we obtain

$$\frac{3}{2m} A h_3 > 0 ,$$

from which it is clear that $h_3 > 0$.

To obtain real values of *A* in (32) it is necessary to require that $h_1 + 2mn_0 < 0$, which can be achieved for $h_1 < 0$ and $|h_1| > 2mn_0$.

Apparently, the first of these conditions corresponds to a section $F_C(V)$ with a negative slope, and the second to a violation of the conditions of dynamic stability of the system at low oscillation speeds (dq/dt).

In addition to the considered frictional self-oscillations of the quasi-harmonic type, relaxation self-oscillations can be excited in the system, accompanied by stops in each oscillatory cycle.

Suppression of frictional self-oscillations is a very important engineering task, since they make it difficult to precisely stop the working element and disrupt the smoothness of its movements [6, 7].

The structure of the mathematical model in the MATLAB/Simulink package is built on the basis of the model of a two-mass electromechanical system (Fig. 2) with the addition of a structure for implementing resistance on the second mass, depending on the speed and the «sticking» effect [1, 2]. The parameters of the model presented in Fig. 3 are reduced to translational motion for ease of consideration. Consequently, we have two masses (m_1, m_2), connected by a spring with stiffness C_{12} without internal losses, which move with velocities V_1 and V_2, respectively. The driving force of the first mass F_1 is formed by a structure reflecting the electromechanical properties of the engine with the stiffness of the mechanical characteristic γ (the electromagnetic time constant is assumed to be zero). The output of the **Integrator2** block is the speed of the second mass. The logic for implementing the «sticking» and «breakdown» effects is discussed in detail in [8]. Thus, the **Integrator2** block has additional inputs (for setting the initial conditions and the impulse for establishing these conditions) and an information output (at the top of the block), which has an output signal V_2. At the output of the **Switch** block, we have the drag force F_C, which is either equal to the

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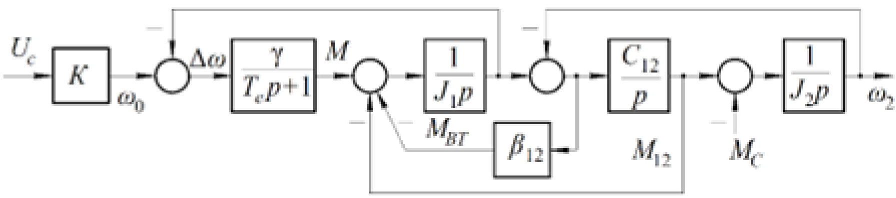

Fig. 2. Unified structural diagram of a two-mass electromechanical system [1, 2]

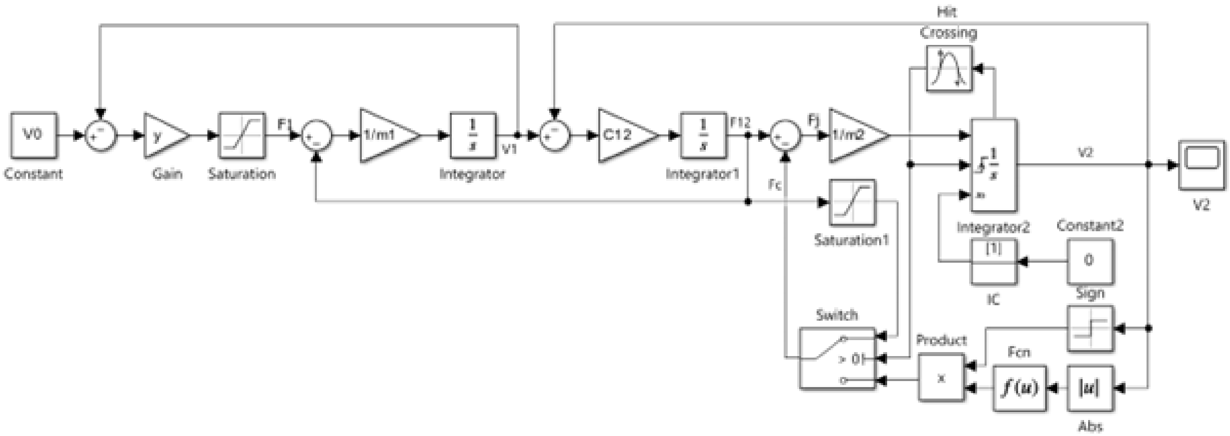

Fig. 3. Mathematical model of a mechanical system for studying frictional self-oscillations [1]

driving force (in the case when the second mass is stationary, and the driving force F_{12} is less than the shear force), or as a function of the velocity V_2 when body 2 is moving. The switching signal is the signal from the zero crossing block (**Hit Crossing**). The switching level of the **Switch** block is set to 0.5. The «shear» force of body 2 (F_S) is specified in the **Saturation1** block. The dependence of the drag force on the velocity F_C (V_2) is specified analytically in the **Fcn** block

Consequently, when applying a driving force F_{12} at $V_2 = 0$, we will have a signal equal to F_{12} at the output of the **Switch** block, while the dynamic force F_j (at the output of the **Sum** block) will be zero, and body 2 will be motionless. When the force F_{12} exceeds the value of the shear force F_S, at the output of the **Saturation1** and **Switch** blocks we have a force $F_S < F_{12}$, and at the output **Sum** a positive value of the dynamic force, under the action of which body 2 will begin to move. When body 2 shifts, a signal will appear at the output of the **Hit Crossing** block, which will reverse the signal from the **Product** block to the **Switch** output – the resistance force will be formed according to the dependence specified in the **Fcn** block. Similar processes occur when body 2 comes to a stop.

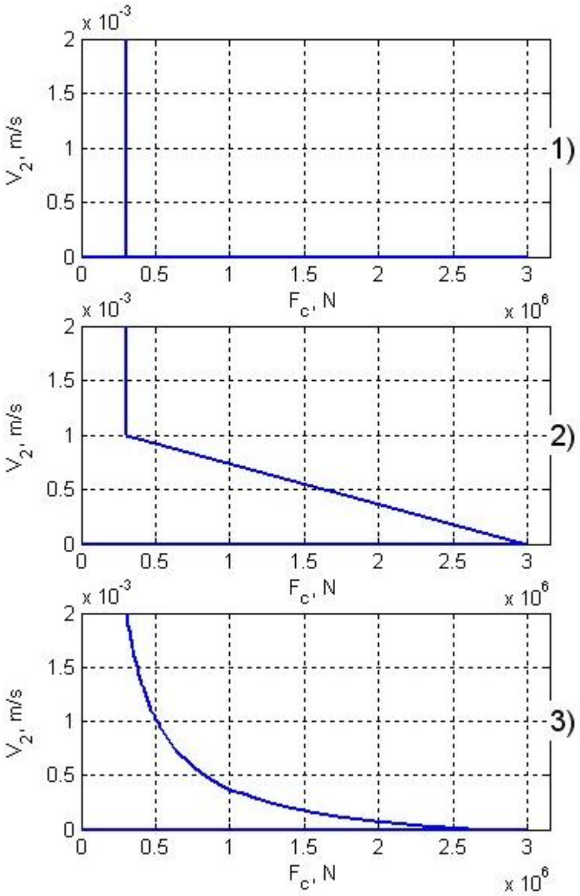

Fig. 4. Forms of the studied load characteristics

The model was used to study an electromechanical system with the following parameters:

$$m_1 = 0{,}1\cdot 10^8 \text{ kg};\ m_2 = 0{,}5\cdot 10^8 \text{ kg};$$

$$C_{12} = 1\cdot 10^{11} \text{ N/m};\ \gamma = 1{,}5\cdot 10^{10} \text{ Ns/m}.$$

Shear force $F_S = 3\cdot 10^6$ N.

The processes of development of self-oscillations in the considered scheme were obtained at a given speed of 1 mm/s for each of the load characteristics (Fig. 4). Fig. 5 shows the velocities of the first and second masses, the elastic force (F_{12}) and the resistance force on the second mass (F_{C2}) and the dynamic force F_j.

The results of this study on the influence of nonlinearities on parametric oscillations of electromechatronic systems can be directly applied in the synthesis and tuning of adaptive and robust control systems. The identified patterns of occurrence of parametric instability zones and the conditions for the development of self-oscillatory modes provide a theoretical basis for the development of control laws aimed at actively limiting oscillation amplitude and preventing the loss of dynamic stability.

Within the framework of adaptive control, modeling results can be used to develop algorithms for automatically adjusting controller parameters, taking into account the current system state, the level of oscillations, and changes in mechanical characteristics, particularly friction parameters and the stiffness of elastic connections. This improves control accuracy and reduces the system's sensitivity to parametric disturbances.

The use of robust control based on the research presented in this article ensures the continued operability of electromechatronic systems under parameter uncertainty and the presence of external disturbances. Taking into account nonlinear effects and parametric oscillations during the synthesis of robust controllers allows for an expansion of the dynamic stability range and increased system reliability in boundary operating conditions.

Thus, the obtained results complement and develop the provisions presented in the works of the authors [9-10], and can be used as a methodological basis for the synthesis of computer-integrated adaptive and robust control systems for electromechatronic objects with increased requirements for dynamic stability, accuracy and operational reliability.

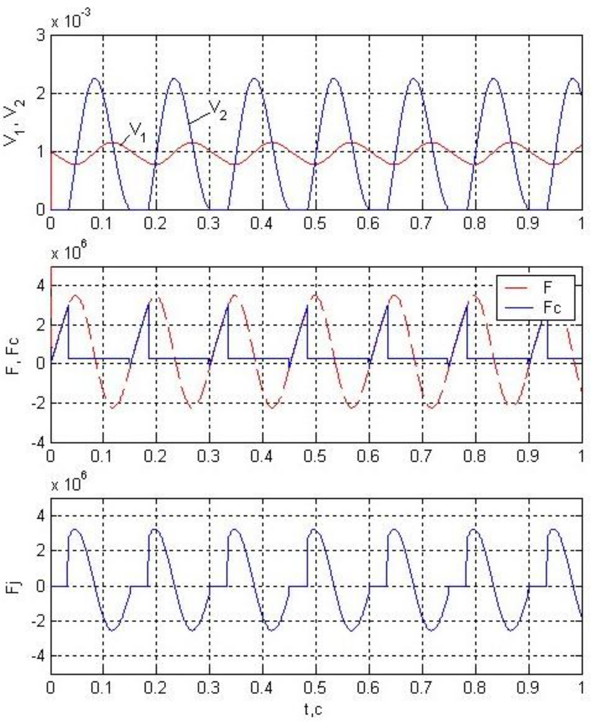

Fig. 5, a. The process of development of self-oscillations at the moment according to Fig. 4 (1)

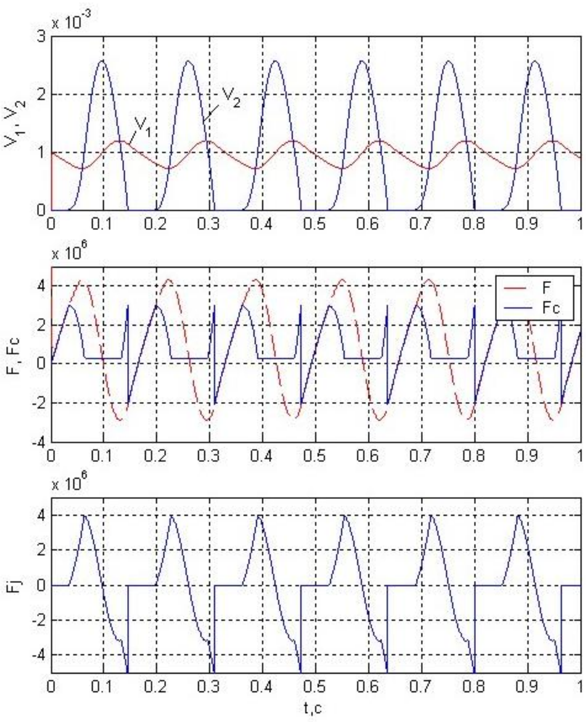

Fig. 5, b. The process of development of self-oscillations at the moment according to Fig. 4 (2)

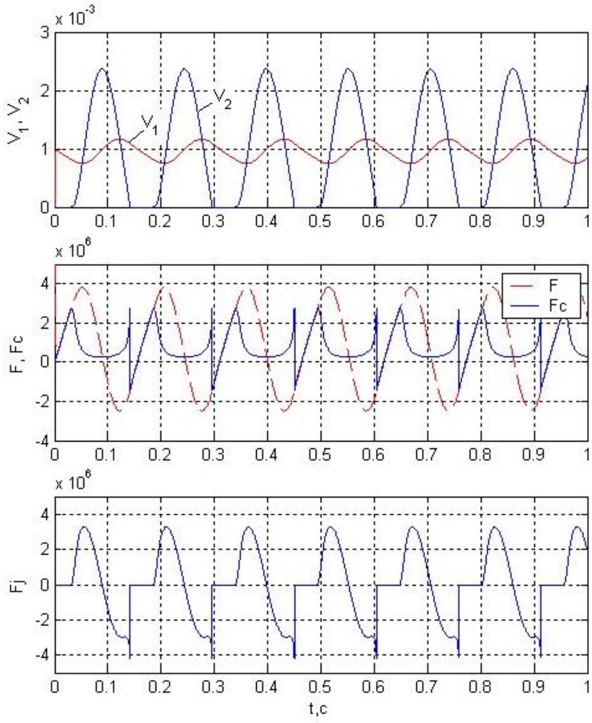

Fig. 5, c. The process of development of self-oscillations at the moment according to Fig. 4, (3)

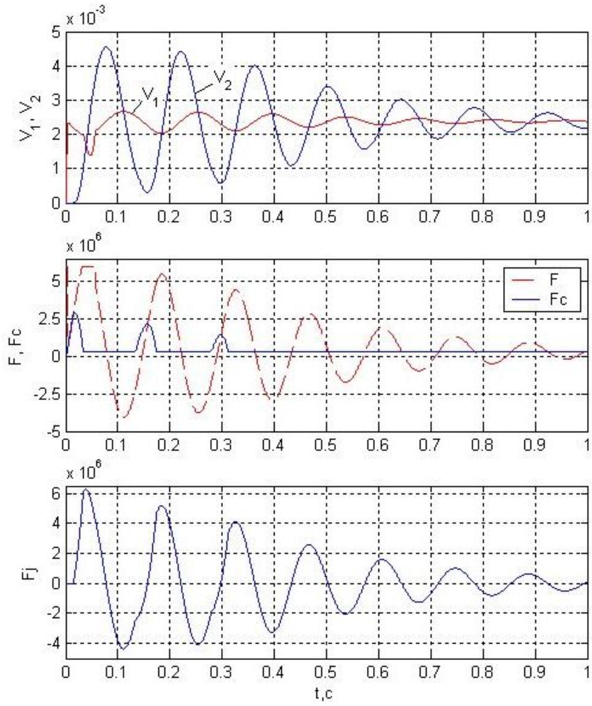

Fig. 5, d. Limit mode of breakdown of self-oscillations at V_1 = 0.024 m/s and the moment of resistance according to Fig. 4, (2) The implementation of computer-integrated adaptive and robust control systems based on the research into nonlinear effects and parametric oscillations presented in this article ensures the alignment of the dynamic modes of an electromechatronic system with the requirements of a specific technological process. This reduces the impact of unwanted oscillatory modes, improves the stability and control accuracy of actuators, and leads to process optimization, reduced energy losses, and increased overall efficiency of electromechatronic systems.

Conclusions. This article examines the influence of nonlinear characteristics of electromechatronic system elements on parametric oscillations and their key dynamic parameters. The analysis revealed that the presence of nonlinearities (magnetic circuit saturation, backlash and dry friction, elastic nonlinear characteristics, current and voltage limitations) significantly alters the conditions for the occurrence and development of parametric resonance.

It has been established that parametric oscillations are one of the most dangerous forms of dynamic instability, as they develop due to periodic or quasi-periodic changes in system parameters, rather than as a result of an external disturbance. Under such conditions, even small initial disturbances or model inaccuracies can lead to an exponential increase in oscillation amplitude and a loss of system control.

Using the conditional oscillator method, analytical relationships have been derived that allow us to describe the development of parametric resonance and formulate sufficient conditions for dynamic stability, taking into account nonlinear effects. It has been shown that nonlinearity can either expand or contract the system's instability regions, depending on their type and intensity. Incorporating nonlinear factors into mathematical models allows for more accurate predictions of the actual operating modes of electromechatronic systems, particularly under conditions of variable power supply or load parameters.

The processes of development of self-oscillations in the circuit at a given speed of 1 mm/s for three types of load characteristics were obtained, which made it possible to obtain the dependences of the speeds of the first and second masses, the elastic force and the resistance force on the second mass and the dynamic force on time.

Special attention is given to the analysis of frictional self-oscillations arising in systems with negative effective viscous friction. It is shown that such oscillations form after passing through a zone of dynamic instability and can degrade positioning accuracy, smoothness of motion, and the operational reliability of electromechatronic systems. The resulting analytical conditions for the existence and stability of self-oscillatory modes are consistent with the physical nature of sliding friction processes.

A mathematical model of a two-mass electromechanical system, implemented in MATLAB/Simulink, allowed us to study the development of self-oscillations for various load characteristics. The modeling results confirmed the significant influence of the shape of the drag force characteristic on the dynamic behavior of the system and demonstrated the possibility of both persistent self-oscillations and their ultimate breakdown modes.

The obtained results confirm the need to use nonlinear models in the design and modernization of control systems, as simplified linear approaches can lead to underestimation of the risk of undesirable resonant modes. The practical significance of the study lies in the possibility of targeted selection of system parameters and control laws to improve dynamic stability, reliability, and operational efficiency.

The authors' subsequent research will focus on developing computer-integrated control systems for electromechatronic systems capable of ensuring stable operation under nonlinear conditions, parametric oscillations, and variable load conditions. Integrating mathematical models of nonlinear dynamics with real-time hardware and software controls will improve the accuracy of detecting and preventing hazardous resonant modes.

A promising approach is the use of adaptive and robust control, which ensures the system's dynamic stability under parametric uncertainty, external disturbances, and changes in the characteristics of the mechanical and electromagnetic subsystems of an electromechanical system. The combination of adaptive algorithms and robust methods will allow automatic reconfiguration of control laws and limit the development of parametric and frictional self-oscillations.

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Руднєв Є. С., Романченко Ю. А. Дослідження впливу нелінійностей на параметричні коливання в електромехатронних системах.

В статті представлений аналіз параметричних коливань в електромехатронних системах та вплив на них нелінійностей, обумовлених фізичними властивостями елементів і конструктивними особливостями системи.

Показано, що реальні електромехатронні системи характеризуються наявністю різноманітних нелінійностей. Факторами, що роблять механічну систему нелінійною, можуть бути нелінійні сили, які виникають при роботі такої системи, та її перемінні параметри. Встановлено, що особливе місце серед динамічних явищ, які виникають в електромехатронних системах, займають параметричні коливання, зумовлені періодичною або квазіперіодичною зміною параметрів системи в часі.

В статті розглянуто ефект параметричного резонансу, що є проявом динамічної нестійкості системи, коли малі збурення можуть призвести до суттєвих змінень руху системи. Отже будь яке випадкове збурення на достатньо великому відрізку часу може призвести до аварійних наслідків. Нелінійні фактори в механічній системі з параметрами, що періодично змінюються, проявляються, в основному, в зонах параметричних резонансів.

Авторами подано поняття автоколивань, що можуть виникати у нелінійних системах при відсутності періодичної збурюючої сили та, як приклад, розглянуто квазігармонічні фрикційні автоколивання. Подавлення фрикційних автоколивань є дуже важливою інженерною задачею, оскільки вони затрудняють точну зупинку робочого органу та порушують плавність його переміщень.

Наведено уніфіковану структурну схему для будь-якого типу електромеханічної системи електропривода, її параметри, а також математичну модель.

Виконано моделювання електромеханічної системи в середовищі MATLAB/Simulink відповідно до структурної схеми із заданими параметрами.

Отримано процеси розвитку автоколивань у розглянутій схемі при заданій швидкості для трьох варіантів досліджених характеристик навантаження.

Наведено залежності швидкостей першої та другої мас, пружної сили та сили опору на другій масі і динамічної сили від часу. Встановлено, що реалізація комп'ютерно-інтегрованих адаптивних і робастних систем керування з урахуванням наведених досліджень нелінійних ефектів і параметричних коливань забезпечує узгодження динамічних режимів електромехатронної системи з вимогами конкретного технологічного процесу. Це дозволяє зменшити вплив небажаних коливальних режимів, підвищити стабільність і точність керування виконавчими механізмами.

Ключові слова: *параметричні коливання, електромехатронні системи, нелінійність, математична модель, збурення, автоколивання, синтез, комп'ютерно-інтегрована система керування, адаптивне керування, робастне керування.*

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