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# AXONOMETRIC TRANSFORMATIONS OF A COMPLEX DRAWING

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# АКСОНОМЕТРИЧНІ ПЕРЕТВОРЕННЯ КОМПЛЕКСНОГО КРЕСЛЕНИКА

**Карпюк Л. В., Давіденко Н. О., Гезеві Абдалхалех Гома Ахмед**

*When drawing up technical drawings, it is sometimes necessary to have visual representations along with images of objects in the orthogonal projection system. For such images, the method of axonometric projection is used (axonometry is a Greek word, literally translated as measurement along the axes; axon means axis, metreo means measuring). The essence of the axonometric projection method: an object, together with the axes of rectangular coordinates to which it is assigned in space, is projected onto a certain plane so that none of its coordinate axes is projected onto it at a point. In this case, the object itself is projected onto this projection plane in three dimensions. In this case, the dimensions of the object in axonometric projections along all three axes are distorted. The change in linear dimensions along the axes is characterized by the distortion indices (coefficients) along the axes. The distortion index is the ratio of the length of a segment on the axonometric axis to the length of the same segment on the corresponding axis of the rectangular coordinate system in space. The purpose of this paper is to construct an axonometric projection of a circle located in the plane of general position using classical methods of descriptive geometry - methods of transforming a complex drawing. Axonometry stands alone in descriptive geometry, which is due to the theoretical multitude of approaches in which the coordinate system is a model of geometric space. Certain difficulties in mastering axonometry are related not only to this but also to its insufficient methodological support. Thus, in the definition of axonometric projection, the standards do not contain key phrases about the orientation of the object and the associated coordinate system relative to the projection direction, which should not be parallel to any coordinate axis. There is no clarity about axonometric planes, their location relative to the main planes in rectangular projection. The article describes a method for constructing an axonometric projection of a circle located in the plane of general position relative to the planes formed by axonometric axes. An example of constructing an ellipse - a projection of a circle for a projecting plane - is given, which can be extended to the conditions of the plane of general position without fundamental changes. The proposed article can be recommended for students who study descriptive geometry and engineering graphics courses in depth, and can also be useful for young teachers as a supportive material in working with students.*

***Keywords**: circle, axonometric projection, ellipse, plane of general position, orthogonal coordinate system, axis.*

**Introduction.** Drawings in the rectangular projection system are easy to create and easy to determine the dimensions of objects, but they have a significant drawback: lack of clarity. To make it easier to read a complex drawing and more fully identify the shape of an object, it is often supplemented with an axonometric image. Attention to the theory of axonometric projections is explained by its simplicity and accuracy, because it is easy to determine the true width, height, and depth of the depicted object from an axonometric image.

The essence of the parallel axonometric projection method is that the object is referred to a certain coordinate system and then projected by parallel rays onto the plane along with the coordinate system. The main purpose of using axonometric projections is to build an image that is closer to the way the eye perceives than a drawing in orthogonal projections.

Axonometric projections are widely used in various fields of technology for visual representations, they are a complement and explanation to orthogonal projections, so mastering the techniques of constructing axonometric projections is important for further practical work. Axonometry makes it easier to visualize a pattern, its parts, and their relationships. When constructing axonometry, a part with a coordinate system attached to it is projected in a special way onto the plane of the drawing, and there is a strict unambiguous relationship between the orthogonal drawing and the axonometric image.

**Presentation of the main material.**

In practice, you often have to build axonometric projections of circles whose planes are parallel to any coordinate plane. In the rectangular isometric and dimetric projections, a circle is projected into an ellipse, so it is important to know how their axes pass.

To get a visual representation of the axes, let's write the circles into the faces of the cube, i.e. squares. The circles will touch the edges of the cube in their centers. In Figure 1, you can see that the points of contact of the ellipses into which the circles are transformed are also in the middle of the sides of the rhombuses into which the squares were projected. In addition to these points of contact, it is important to have four more points belonging to the ends of the major and minor axes of the ellipse

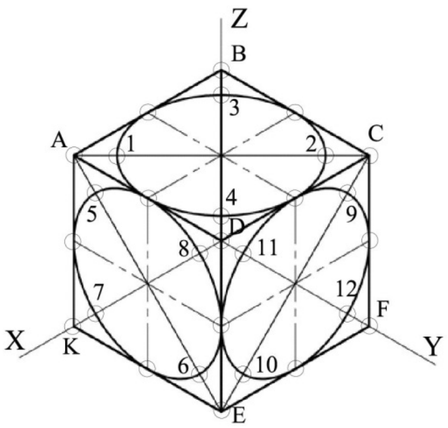


Fig. 1. Axonometric projection of a cube with ellipses inscribed in its faces

In rectangular isometric and dimetric projections, the major axis of the ellipse in any face of the cube is located along the major diagonal of the rhombus, and the minor axis is located along the minor diagonal.

Theoretically, the issue of constructing an axonometric projection of a circle is discussed in full in the educational literature [1, 2, 3, 4, 5].

This paper presents practical methods for constructing axonometric projections of circles located in planes of general position.

In general, a circle is projected onto the axonometric plane into an ellipse whose major axis is the projection of the diameter parallel to the axonometric plane.

The axonometry of any circle can be performed as an ellipse drawn through several axonometric projections of points of a given circle, each of which is found along the coordinate line corresponding to that point. The use of this universal method is limited by the large volume of constructions and very low final similarity.

If a circle is parallel to the axonometric plane, then its axonometric projection is a congruent circle. This special case is used in oblique dimetric axonometry, in which one of the planes of the orthogonal coordinate system is parallel to the axonometric plane.

If the plane passing through the circle is perpendicular to the axonometric plane, then the projection of the circle is a line segment equal to the diameter. Such cases should be avoided, as it reduces the clarity of the image [6].

The construction of axonometric projections of circles located in the planes of an orthogonal coordinate system or in planes parallel to them, for rectangular and oblique axonometric projections, is given in the standards.

For a circle placed in the plane of general position [1], the construction of an ellipse begin by determining the direction of the minor axis, which coincides with the projection of the perpendicular to the plane of the circle onto the axonometric plane. The perpendicular passes through the center of the circle.

The length of the minor axis is obtained as the projection of the radius extending from the center of the circle along the same perpendicular.

The major axis of the ellipse is perpendicular to the minor axis and equal to the diameter of the circle. Consider another way to construct the axonometric projection of a circle when it is located in a plane that is not parallel to the planes of the orthogonal coordinate system.

We will build the axonometry of a circle by first inscribing it in a square. An example of the construction is shown in Figures 2, 3 for the conditions of a rectangular isometric projection (the projection direction is perpendicular to the axonometric plane and the distortion coefficients along the X, Y, Z axes are equal to each other and equal to one).

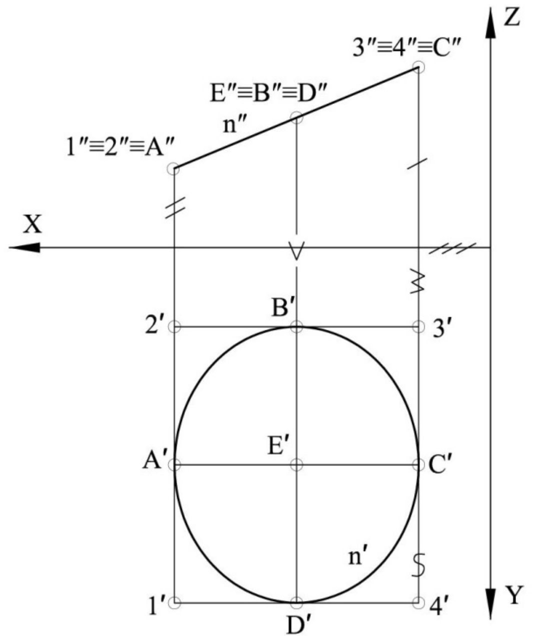


Fig. 2. Construction of axonometry of a circle lying in a plane not parallel to the planes of the orthogonal coordinate system (original complex drawing)

Superimpose the axes of the orthogonal coordinate system on the projection axes of the complex drawing, and place the circle **n** in the frontal projection plane.

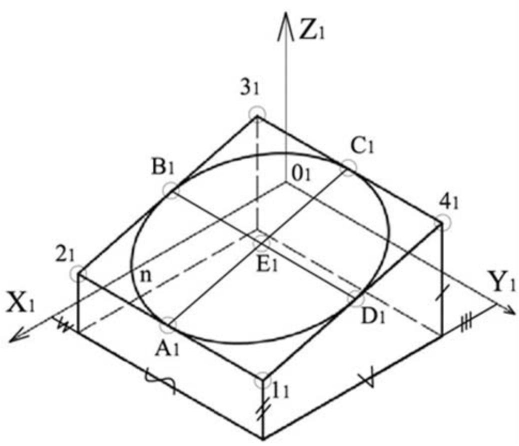


Fig. 3. Construction of the axonometry of a circle lying in a plane that is not parallel to the planes of the orthogonal coordinate system (axonometric)

Around the circle we describe the 1-2-3-4 square (Fig. 2). The constructions in Fig. 3 are based on the initial data of Fig.2. The axonometric projection of the square is the parallelogram $1_1$-$2_1$-$3_1$-$4_1$ (Fig. 3). The image of the parallelogram is made due to the equality of coordinates for the points of the complex drawing and their axonometry ($k_x$=$k_y$=$k_z$=1). Dividing the sides of the parallelogram in half, we get four points of the ellipse. As a first approximation, the ellipse can be drawn through these points, as well as under the condition of touching the sides of the parallelogram at these points and the known form of this second-order curve.

To construct intermediate points, a scheme can be used (Fig. 4), when the conjugate diameters are taken as the initial data. When projecting a circle into an ellipse (when the circle is compressed), two perpendicular diameters are mapped into two ellipse diameters, which are called conjugate diameters.

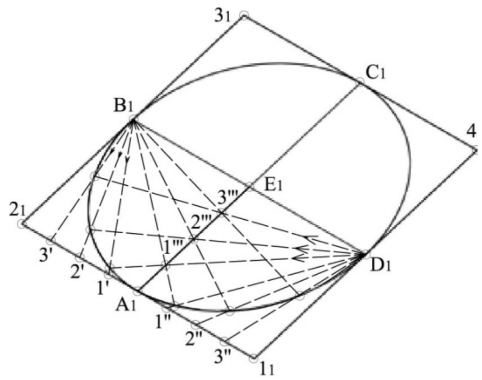


Fig. 4. Construction of an ellipse by given connected diameters

The half-sides of the parallelogram $2_1A_1$; $A_11_1$ are divided into equal segments by the points 1'; 2'; 3'; 1''; 2'; 3'' in the specified sequence. Also, create equal segments $E_1$3'''; 3'''2'''; 2'''1'''; 1'''$D_1$ on the half of the connected diameter. The points of the ellipse are located at the intersection of the rays coming from $B_1$; $D_1$ and passing through the dividing points with the same numbers, as shown in Fig. 4.

In axonometric projections, when constructing an ellipse manually, the pattern curve is replaced by a four-center oval consisting of four consecutive arcs of circles.

For our case, the direction and lengths of the major and minor axes of the ellipse are determined using the known axonometric projections of the connected diameters $A_1C_1$ and $B_1D_1$ [4]. These data are the initial data and are sufficient to construct the ellipse.

For circles located in the planes of general position with respect to the planes defined by axonometric axes, without any significant changes, we apply the above method of constructing axonometric projections of a circle.

Next, let's look at the method of constructing an ellipse - an axonometric projection of a circle at eight points. This method is advisable to use when the diameter of the circle is less than 15-20 mm.

In a complex drawing book, we enclose the circle in a square (Fig. 1) and build its axonometric projection - a rhombus or parallelogram (Fig. 5).

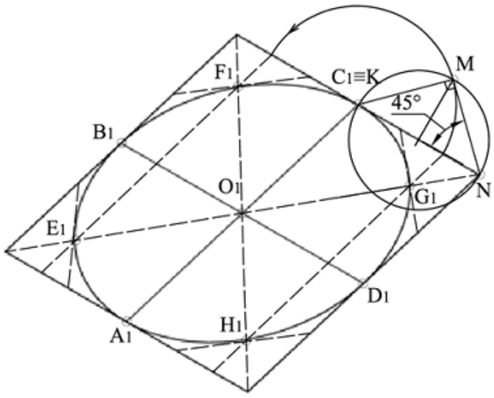


Fig. 5. Construction of an ellipse by eight points

At the intersection of the diagonals we find the center of the ellipse and through this point we draw lines parallel to the sides of the quadrilateral, on which the points of the ellipse $A_1B_1C_1D_1$ are located.

Any half of the side of the quadrilateral is the hypotenuse of the right isosceles triangle KMN. The triangle's catenary KM is placed as shown in Fig. 3 on the side of the quadrilateral.

From the resulting notches, we draw lines parallel to $A_1C_1$ and at the intersection with the diagonals we find four more points of the ellipse $E_1F_1G_1H_1$. Through these points, additionally using the conditions of contact, we draw the ellipse by hand. Additional tangents passing through the points $E_1$, $F_1$, $G_1$, $H_1$ are parallel to the diagonals. An ellipse is a smooth monotonous curve (no breakpoints or jumps in the radius of curvature) with a center of symmetry and two axes of symmetry. The legitimacy of such constructions can be proved under the conditions of parallel design. [5].

If a given part has four or more identical small-diameter circles, it is justified to make a template, which is then fixed in the right places on the drawing with two pins and traced. Cardboard is suitable as a template material.

For rectangular isometric projection when making small ellipses, it is allowed to use ready-made industrial stencils with ellipse-shaped holes

**Conclusions.** The proposed method of constructing an axonometric projection of a circle belonging to the plane of general position is simple and versatile. It does not require knowledge of additional specific techniques, provides the necessary accuracy, and in all cases, it is possible to replace the ellipse with a four-centered oval. Such an approach to solving this problem can be recommended for students who study descriptive geometry and engineering graphics courses in depth, and can also be useful for young teachers as a support material in working with students.

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**Карпюк Л. В., Давіденко Н. О., Гезеві Абдалхалех Гома Ахмед. Аксонометричні перетворення комплексного креслення**

*При складанні технічних креслеників іноді виникає необхідність поряд із зображеннями предметів у системі ортогональних проєкцій мати наочні зображення. Для таких зображень застосовують метод аксонометричного проєктування (аксонометрія - грецьке слово, у дослівному перекладі воно означає вимір по осях; аксон - вісь, метрео - вимірюю). Сутність методу аксонометричного проєктування: предмет разом з осями прямокутних координат, до яких він віднесений в просторі, проєктується на деяку площину так, що жодна з його координатних осей не проєктується на неї в точку.*

*У такому разі сам предмет спроєктується на цю площину проєкцій у трьох вимірах. При цьому розміри предмета в аксонометричних проєкціях по всіх трьох осях спотворюються. Зміна лінійних розмірів вздовж осей характеризується показниками (коефіцієнтами) спотворення вздовж осей. Показник спотворення - це відношення довжини відрізка на аксонометричній осі до довжини такого ж відрізка на відповідній осі прямокутної системи координат в просторі. Мета ж даної роботи – побудова аксонометричної проєкції кола, розташованого в площині загального положення, використовуючи класичні методи нарисної геометрії – способы перетворення комплексного кресленика. Аксонометрія стоїть окремо в нарисній геометрії, що пов'язано з теоретичною безліччю підходів, при якій система координат є моделлю геометричного простору. Певні труднощі у освоєнні аксонометрії пов'язані не тільки з цим, але й з недостатнім її методичним забезпеченням. Так у визначенні аксонометричної проєкції в стандартах немає ключових фраз про орієнтацію об'єкта та пов'язаної з ним системи координат щодо напрямку проєктування, який не повинен бути паралельним жодній координатній осі. Відсутня чіткість щодо аксонометричних площин, їх розташування відносно основних площин при прямокутному проєктуванні. У статті викладено спосіб побудови аксонометричної проєкції кола, розташованого в площині загального положення стосовно площин, утворених аксонометричними осями. Наведено приклад побудови еліпса – проєкції кола для проєкцюючої площини, котрий без принципових змін може бути поширений і для умов площини загального положення. Пропонована стаття може бути рекомендована для студентів, які поглиблено вивчають курси нарисної геометрії та інженерної графіки, а також може бути корисною для молодих викладачів-початківців як допоміжний матеріал у роботі зі студентами.*

***Ключові слова**: коло, аксонометрична проєкція, еліпс, площина загального положення, ортогональна система координат, вісь.*

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